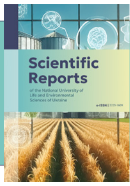
Received 05.08.2025 Revised 03.11.2025 Accepted 27.11.2025

Vol. 21, No. 6
2025



Scientific Reports of the National University of Life and Environmental Sciences of Ukraine

Journal homepage: https://scireports.com.ua/en

UDC 514.18

DOI: 10.31548/dopovidi/6.2025.31

Formation a strip with a trapezoidal cross-section into a ring by rolling between conical rolls

Tetiana Kresan
PhD in Technical Sciences, Associate Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0002-8280-9502

Tetiana Volina*
Doctor of Technical Sciences, Associate Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0001-8610-2208

Victor Nesvidomin
Doctor of Technical Sciences, Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0002-1495-1718

Tetyana Fedoryna
PhD in Technical Sciences, Associate Professor
National University of Life and Environmental Sciences of Ukraine
03041, 15 Heroiv Oborony Str., Kyiv, Ukraine
https://orcid.org/0000-0002-8854-3768

Taras Pylypaka
PhD in Technical Sciences, Associate Professor
National University of Water and Environmental Engineering
33028, 11 Soborna Str., Rivne, Ukraine
https://orcid.org/0009-0000-5582-1859

Suggested Citation:
Kresan, T., Volina, T., Nesvidomin, V., Fedoryna, T., & Pylypaka, T. (2025). Formation a strip with a trapezoidal cross-section into a ring by rolling between conical rolls. *Scientific Reports of the National University of Life and Environmental Sciences of Ukraine*, 21(6), 31-42. doi: 10.31548/dopovidi/6.2025.31.

**Corresponding author*

Abstract. The formation of helical surfaces requires a ring that enables their continuous production without welded seams. The aim of this study was to develop an analytical description of the process of deforming a straight strip into a ring as a result of rolling it between conical rolls. The research methodology was based on analytical modelling of the process of plastic deformation of the strip using assumptions about the isotropy of the material and the constancy of its volume. As a result of the study, analytical dependencies were obtained between the dimensions of the strip cross-section before rolling and the parameters of the ring after rolling, which allow determining the optimal geometric characteristics for forming rings of a given radius. Generalised analytical expressions were developed to determine the elongation coefficient and ring radius, which take into account the change in the aspect ratio of the trapezoidal cross-section of the strip. A model of step-by-step rolling has been proposed, which describes the gradual reduction of the aspect ratio of the trapezoidal cross-section to a rectangular one, ensuring a uniform cross-section of the ring. The obtained expressions made it possible to solve both direct and inverse problems, determine the parameters of the initial strip for forming rings of different radii, predict the geometry of the ring based on the known dimensions of the strip, and use the obtained data to adjust the rolls before experimental verification. The proposed model of step-by-step rolling ensured a uniform cross-section of the ring, which increases the accuracy of manufacturing screw surfaces without welded seams. The results may be applied in mechanical engineering for the production of screws, helical surfaces, and conveyors without welded seams, particularly in the agricultural, food-processing, construction, and energy industries

Keywords: plastic deformation; volume; isotropic material; ring radius; relative elongation

Introduction

Deformation of a ring with a rectangular cros-section can form a helical surface, which is widely used in various fields of mechanical engineering. There were various technologies for manufacturing helical surfaces, including welding individual turns. This method has limitations due to the presence of welded joints, which reduce the strength of the structure. When forming a flat ring by rolling, it is possible to manufacture solid helical surfaces of the required length without welded joints, which increases their reliability. Existing technologies for winding helical surfaces from a straight strip with a rectangular cross-section require considerable effort to deform the material. In addition, this manufacturing method often leads to instability of the deformation process, which can cause metal breaks on the outer edge and the formation of corrugations on the inner edge. Thus, the manufacture of helical surfaces from a ring, which is an approximate development of the surface, is simpler, more accurate and more cost-effective. That is why the formation of a ring of a given radius by rolling a straight strip remains a relevant problem that requires in-depth research.

Helical surfaces are an integral part of many devices and mechanisms, in particular for transporting bulk materials, soil cultivation and the production of high-precision parts. Despite their widespread use, existing manufacturing methods have limitations, particularly in terms of thickness uniformity and geometric accuracy. I. Hevko *et al*. (2021) proposed using elastic helical surfaces to prevent damage to bulk materials during transport. The authors focused on modifying the material but did not consider optimising the geometry of the blank for forming the ring, which limits the application of the method for heavy-duty structures. M.B. Klendii & A.P. Dragan (2021) investigated the use of helical surfaces for soil cultivation, focusing on the design features of the working bodies, but did not address the issue of forming blanks with a trapezoidal cross-section, which could improve the strength characteristics of the parts. Various approaches, including both analytical and numerical methods, have been proposed to solve the problem of optimising surfaces that pass through a spiral curve with a variable pitch. For example, A. Nesvidomin *et al.* (2025) developed a method for optimising the model of a developing surface for such cases, which can be applied to rolling and surface forming processes in various fields of mechanical engineering.

Scientists O. Liashuk *et al.* (2019) conducted a technical and economic justification for the manufacture of screw working bodies, but did not propose analytical models for predicting the geometric parameters of rings during rolling. I. Oh *et al.* (2019) investigated defects that arise during the rolling of L-shaped rings and proposed methods for minimising them, but did not consider the forming of rings with a trapezoidal cross-section, which ensures a uniform thickness of the finished part. L. Liang *et al.* (2021) focused on the accuracy criteria for rolling rings with a profiled cross-section, and L. Liang *et al.* (2022) focused on the mechanisms of geometric defect formation. Although these works describe the deformation processes in detail, they do not offer analytical dependencies for calculating the parameters of the initial strip, which limits the possibility of optimising the process.

Researchers D. Bai *et al.* (2021) analysed intelligent forming technologies but did not address the specifics of rolling trapezoidal strips, which allows avoiding ring thickness irregularities. S. Yuan & X.B. Fan (2019) outlined the prospects for the precise forming of ultra-large integrated components, but did not consider the step-by-step rolling of strips with a trapezoidal cross-section, which allows the geometry of the ring to be controlled at each stage of deformation.

To summarise the above, the following general, previously unresolved issues can be formulated:

- the lack of analytical models for calculating the parameters of the trapezoidal cross-section output strip required to form rings of a given radius;
- insufficient attention by researchers to step-by-step rolling, which ensures a uniform cross-section of the ring and reduces the risk of defects;
- limited information on optimising the settings of conical rolls for forming rings with high precision.

It should be noted that the issue of forming helical surfaces from flat blanks can be considered from the point of view of the rational selection of these blanks. A flat ring can be cut from sheet material, but this method leads to significant material waste and costs for cutting the required contour from it. It is much easier to obtain this contour by rolling a straight strip. When forming a ring by rolling a strip of rectangular cross-section, it will have a variable thickness, which will be thinner at the periphery of the ring, i.e. at the outer edge of the screw surface, which wears out the most. In order for the thickness of the ring to be uniform, a straight strip with a trapezoidal cross-section must be used.

The aim of the study was to provide an analytical description of the transformation of a straight strip with a trapezoidal cross-section into a flat ring of a given radius by rolling it between conical rolls. This raises the following objectives:

- based on the plastic properties of the metal and the geometric properties of the shape, develop an approximate model of the redistribution of its mass under the action of deforming forces;
- find the dimensions of the cross-section of a straight strip for forming a flat ring of a given thickness and radius.

Materials and Methods

The initial thickness H_0 of a straight strip with a rectangular cross-section after rolling between cylindrical rolls (Fig. 1) decreased to a value H_1. It was assumed that the volume of the strip after rolling remained constant. The second condition was that the strip material was isotropic, so elongation occurred equally in all directions. An elongation coefficient p was introduced as the ratio of the corresponding sides of the strip element before and after deformation. In this case, the dimensions of the strip element after deformation were recorded taking into account this coefficient p: $L_1 = p \cdot L_0$, $S_1 = p \cdot S_0$. The thickness of the strip before deformation H_0 and after deformation H_1 were given values. Equating the volumes of the strip element before and after rolling gave the result $L_0H_0S_0 = p^2L_0S_0H_1$, hence:

$$p = \sqrt{\frac{H_0}{H_1}}. \quad (1)$$

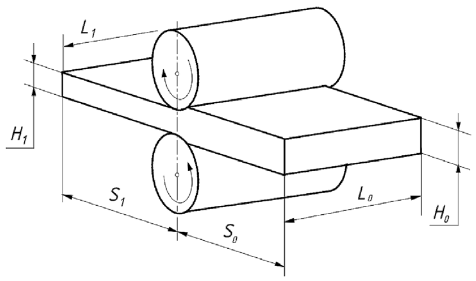


Figure 1. *Diagram of rolling a straight strip with a rectangular cross-section between cylindrical rolls*

Note: H_0 – strip thickness before deformation; H_1 – strip thickness after deformation; L_0 – strip width before deformation; L_1 – strip width after deformation; S_0 – strip length before deformation; S_1 – strip length after deformation

Source: developed by the authors

With a known coefficient p, it is possible to find the longitudinal and transverse elongation of the strip. After rolling, the thickness of the strip decreased, while its width and length changed in proportion to the elongation coefficient. To describe the process, the assumption of material isotropy and volume constancy was used, which allowed the volumes of the strip before and after deformation to be equated. The elongation coefficient was defined as the ratio of the dimensions of the strip element after deformation to its initial values. Based on this, mathematical dependencies were introduced, which were used to further describe the transformation of a straight strip into a ring.

Results

The rolling diagram of a straight strip with a trapezoidal cross-section between conical rolls is shown in Figure 2.

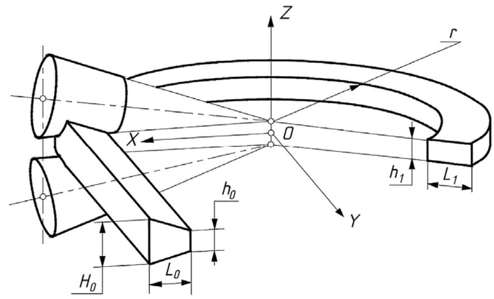


Figure 2. *Diagram of rolling a straight strip with a trapezoidal cross-section into a flat ring*

Note: O_{xyz} – axes of the spatial coordinate system; L_0 – height of the trapezoid of the strip cross-section before deformation; H_0 – larger side of the trapezoid of the strip cross-section before deformation; h_0 – smaller side of the trapezoid of the strip cross-section before deformation; L_1 – width of the strip after deformation; h_1 – thickness of the strip after deformation; r – radius of the inner arc of the ring

Source: developed by the authors

The rolls were adjusted so that the thickness of the ring after rolling the strip was equal to the shorter side of the trapezoid, i.e. $h_1 = h_0$. This means that the intensity of the strip deformation in the transverse direction will increase from the shorter side of the trapezoid to the longer side. Based on this, it can be assumed that during rolling, the material will also elongate in this direction. It follows that the length of the arc of the inner radius of the ring is equal to the length of the corresponding straight segment of the strip before deformation. This means that the outer arc of the ring elongates in the longitudinal direction, while the inner arc does not elongate and transforms into a circle with radius r (Fig. 2).

The volume V_0 of a straight strip element with a trapezoidal cross-section of length s_0 was determined by the following formula:

$$V_0 = \frac{H_0 + h_0}{2} L_0 s_0. \quad (2)$$

After deformation, the element of a straight line will turn into a corresponding element of a flat ring. An image for determining its volume is shown in Figure 3.

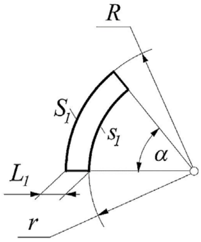


***Figure 3.** Projection of a flat ring element with its dimensions marked*

Note: r – radius of the inner arc of the ring; R – radius of the outer arc of the ring; s_1 – length of the inner arc; S_1 – length of the outer arc; L_1 – width of the strip after deformation; α – central angle

Source: developed by the authors

Given that the thickness of the ring is $h_1 = h_0$, its volume can be found using the formula:

$$V_1 = \frac{R^2 - r^2}{2} h_0 \alpha. \tag{3}$$

It is assumed that the length of the inner arc s_1 did not change during deformation and is equal to the length of the straight strip element, i.e. $s_1 = s_0$. From this, an expression for the angle α can be found: $\alpha = s_0/r$. The length of the outer arc S_1 has increased, as has the width of the strip. The elongation of the strip width can be written in terms of the elongation coefficient p: $L_1 = p \cdot L_0$. In this case, for the radius R of the outer arc, the expression: $R = r + p \cdot L_0$ can be written. The length of the arc S_1 within the central angle α can be found using the known formula: $S_1 = R \cdot \alpha = (r + p \cdot L_0)\ \alpha = (r + p \cdot L_0)\ s_0/r$. On the other hand, the length of the arc S_1 is the result of the elongation of the straight line strip, i.e. $S_1 = p \cdot s_0$. Equating these two expressions, as well as the volume (2) before deformation and the volume (3) after deformation, allows to obtain a system of two equations with two unknown quantities p and r:

$$\begin{cases} \frac{H_0 + h_0}{2} L_0 s_0 = \frac{h_0 s_0}{2r}\left[(r + pL_0)^2 - r^2\right]; \\ p s_0 = (r + pL_0)\frac{s_0}{r}. \end{cases} \tag{4}$$

Reducing equations (4) by s_0 and solving for r and p gives the result:

$$p = \frac{-h_0 + \sqrt{h_0(5h_0 + 4H_0)}}{2h_0}, \tag{5}$$

$$r = L_0 \frac{h_0 + 2H_0 + \sqrt{h_0(5h_0 + 4H_0)}}{2(H_0 - h_0)}. \tag{6}$$

In the case where $H_0 = h_0$, i.e. in the absence of plastic deformation, from (5) $p = 1$, can be obtained, and from (6) $r = \infty$, i.e. there is no elongation and the strip remains straight. The expressions obtained make it possible to solve the inverse problem, i.e. to find the dimensions of the cross-section of a straight strip to obtain a ring of a given radius. From equation (6), an expression for H_0 can be found:

$$H_0 = h_0 \frac{L_0 r - L_0^2 + r^2}{(L_0 - r)^2}. \tag{7}$$

A model of step-by-step rolling of the strip can be developed, i.e. gradually reducing the larger side of the trapezoidal cross-section. Then the strip turns into a ring, the cross section of which will also be a trapezoid, but with a smaller difference between the longer and shorter sides (Fig. 4). In this case, the same deformation conditions remain as in the previous case, i.e. $h_1 = h_0$.

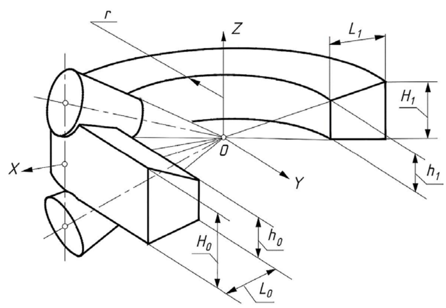


***Figure 4.** Diagram of rolling a straight strip with a trapezoidal cross-section into a ring with a smaller ratio of opposite sides of the trapezoid*

Note: O_{xyz} – axes of the spatial coordinate system; L_0 – height of the trapezoidal cross-section of the strip before deformation; H_0 – larger side of the trapezoidal cross-section of the strip before deformation; h_0 – the smaller side of the trapezoid of the strip cross-section before deformation; L_1 – the height of the trapezoid of the strip cross-section after deformation; H_1 – the larger side of the trapezoid of the strip cross-section after deformation; h_1 – the smaller side of the trapezoid of the strip cross-section after deformation

Source: developed by the authors

The volume of the ring can be found by means of integral calculus as a body formed by the rotation of a trapezoid. Similarly, a system of two equations was obtained, the solution of which gave the following results:

$$p = \frac{-2h_0 - H_1 + \sqrt{16h_0^2 + 28H_1 h_0 + H_1^2 + 12H_0(h_0 + 2H_1)}}{2(h_0 + 2H_1)}, \quad (8)$$

$$r = L_0 \frac{4h_0 - H_1 + 6H_0 + \sqrt{16h_0^2 + 28H_1 h_0 + H_1^2 + 12H_0(h_0 + 2H_1)}}{6(H_0 - H_1)}. \quad (9)$$

When $H_0 = H$ there is no deformation, and the values of p and r are similar to the previous case. If in expressions (8) and (9) the value $H_1 = h_0$, in expressions (8) and (9), new expressions are obtained that exactly coincide with (5) and (6). Thus, expressions (8) and (9) are generalised. They describe the process of gradual formation of a flat ring by reducing the ratio of the larger and smaller bases of an isosceles trapezoid to unity, i.e. to a rectangular cross-section. At the same time, the height of the trapezoid increases, which ultimately turns into the width of the ring.

Let the straight transverse cross-section of the strip have the following dimensions in millimetres: $H_0 = 15; h_0 = 5; L_0 = 40$. The elongation coefficient p can be found according to (5): $p = 1{,}56$. According to (6), the inner radius of the flat ring is $r = 111$. The width of the ring is accordingly $L_0 = p \cdot L_0 = 62$. The dimensions of the cross-section of the ring are 5×62. If the ring needs to be closed, the inner arc s must have a length of $2 \cdot \pi \cdot r = 697$. The straight strip must have the same length, since, according to the condition, the length of the inner arc of the ring is equal to the length of the strip. If $r = 111$, is substituted into formula (7), the result is $H_0 = 15$.

Discussion

In metal pressure processing, a common method is to deform workpieces by rolling them between rolls. Precise analytical results regarding the relative position of the rolls to achieve the desired result are often difficult to obtain due to the complexity of the plastic deformation process. Roll adjustment is usually based on accumulated experience and experimentation. However, to begin with, it is necessary to know at least the approximate value of the radius r (Fig. 2) in order to adjust the rolls correctly. Subsequently, the adjustment can be refined experimentally, but the availability of analytical dependencies significantly reduces the time and cost of equipment preparation.

Unlike the model proposed by L. Hua *et al.* (2023), which focuses on modelling the behaviour of metal during the rolling of thin-walled conical rings, the model proposed in the current study was based on two fundamental properties of plastic deformation of metal: volume constancy and material isotropy. These assumptions allow to obtain analytical dependencies that do not depend on experimental data, unlike the approaches described in the works of Z. Dong *et al.* (2019) and X. Han *et al.* (2023), where the emphasis is on numerical modelling and experimental settings. This is especially important for industrial applications, where the speed of equipment adjustment directly affects production productivity.

It can be seen that for the three rolling cases (Fig. 1, Fig. 2 and Fig. 4), the elongation coefficients p (1), (5) and (8) do not depend on the strip width L_0, but only on the aspect ratio H_0 and H_1. This allows to calculate the dimensions of the transverse trapezoidal cross-section of a straight strip so that, as a result of rolling, flat rings of the same cross-section but with different radii r are obtained. For example, let the dimensions of the straight cross-section of the strip be: $H_0 = 20; h_0 = 5; L_0 = 35$. After its deformation, the value of the rectangular cross-section of the ring will practically not change (5×62), but the radius r will be different: $r = 79$. This indicates the universality of the proposed dependencies, which can be applied to different geometric parameters of the strip. Expressions (8) and (9) are generalised and describe the deformation process of a straight strip with a transverse cross-section that is an isosceles trapezoid. They describe the step-by-step deformation of the strip by reducing the larger side of the trapezoid's base until the bases become equal, i.e. the trapezoid turns into a rectangle. It should be noted that at the intermediate stages of deformation, the vertices of the conical rolls are located in the centre of the ring (Fig. 4), and at the final stage, they are located on the axis of the ring and there is a distance between them equal to the thickness of the ring (Fig. 2). This distinguishes the present study from the work of Z. Dong *et al*. (2019), who focused on the rolling of large conical cylinders, and X. Han *et al*. (2023), who investigated the formation of cylindrical rings with high ribs. Unlike these works, the present study proposes a step-by-step rolling model that describes the reduction of the larger side of the trapezoid until it becomes a rectangle. This is particularly important for controlling the geometry of the ring at intermediate stages of deformation, which was not considered in the works of D.S. Qian *et al*. (2023) and D. Qian *et al*. (2024), where the emphasis was on controlling the rolling process using a closed loop.

It should be noted that X. Chang *et al*. (2023) investigated the influence of roller cross-sectional diameter on the rolling of double-curved surfaces. Like the study described above, this work concerns the processes of rolling and forming complex surfaces using metal pressure treatment methods. X. Chang *et al*. investigated the rolling of double-curved surfaces with a focus on the influence of roller diameter on forming quality. The authors focus on optimising roller parameters to minimise defects. In the study described, the emphasis is on the analytical description of the deformation process and obtaining generalised expressions for calculating the parameters of the strip and ring. This study complements the work of X. Chang *et al.*, proposing a new approach to forming rings from straight strips with a trapezoidal cross-section. Unlike the optimisation of roller parameters for rolling double-curved surfaces, the proposed method allows rings to be obtained directly from the strip, which is more technologically advanced and economical.

The work of W.Y.D. Yuen (2025) focused on the analysis of thermal and mechanical deformations of strips during flat rolling, in particular on the influence of friction, heat generation and uneven stress distribution in the contact zone between the rolls and the strip. The author emphasised the importance of taking into account the thermoelastic deformations of the rolls to prevent defects in the flatness of the strip. As in the study described above, the author analyses the processes of plastic deformation of the material under the influence of external loads, but does not consider the specifics of forming rings with a trapezoidal cross-section, which requires taking into account the uneven elongation across the width of the strip and the gradual reduction of the trapezoid aspect ratio. The study by J. Miao *et al*. (2022) focused on the analysis of contact behaviour and thermal deformations in planetary roller screw mechanisms used to convert rotary motion into linear motion. Unlike the study described above, this work examines the effect of thermal deformations on the accuracy of motion transmission and load distribution in screw mechanisms, as well as the optimisation of tooth profiles to reduce contact stresses and wear. Common to the work of J. Miao *et al.* and the presented study is the emphasis on the importance of considering geometric parameters to achieve the desired characteristics of the final product.

The work of Y. Caoqi *et al.* (2024) is devoted to the issue of synchronous forming of the outer shape and inner hole of hollow shafts using cross-screw rolling with a mandrel. The authors focused on analysing the influence of process parameters (tension angles, roll rotation speed, etc.) on the axial force and the quality of the final shape. Similar to the study by Y. Caoqi *et al.*, they considered the plastic deformation of metal under the influence of external forces, emphasising the importance of the geometric parameters of the workpiece and tool in order to optimise the deformation process and achieve a high-quality final product. Unlike Y. Caoqi *et al.*, the presented study obtained generalised analytical expressions for the elongation coefficient and ring radius, which makes the proposed approach universal and suitable for a wide range of geometric configurations. An innovative method for forming railway car axles using three-roll cross-screw rolling was proposed by Z. Pater *et al.* (2020). The method proposed by the authors allows complex profiles to be formed with high accuracy. In contrast to that work, the present study develops a model that describes the step-by-step reduction of the trapezoidal cross-section aspect ratio to a rectangular form, which allows control of ring geometry at intermediate stages of deformation and minimises the risk of defects.

Scientists S. Pylypaka *et al.* (2022) focused on studying the process of bending flat rings into screw conoids – conical surfaces formed by rotating a straight line around an axis at a certain angle. Like the study described above, the work of S. Pylypaka *et al.* focused on the processes of forming helical surfaces and rings using metal pressure treatment methods. Unlike S. Pylypaka *et al.*, who described the corrugation of the inner edge and the uneven wall thickness of the conoid, the presented study focuses on the uneven thickness of the ring and possible metal breaks. S. Pylypaka *et al.* used an experimental approach and numerical modelling to analyse the bending process. The authors focused on determining the optimal angles of inclination of the helix and material parameters. In contrast, the present study uses analytical modelling based on the properties of plastic deformation (constant volume and isotropy of the material). This allows analytical dependencies to be obtained for calculating the parameters of the strip and ring. Thus, the results obtained complement the research of S. Pylypaka *et al.*, offering a new approach to the formation of rings from straight strips of trapezoidal cross-section. Unlike the bending of finished rings, the proposed method allows rings to be obtained directly from the strip, which is more technological and economical. The analytical dependencies obtained in the work allow optimising the parameters of the strip and ring, which makes the presented approach more universal and accurate.

M. Pylypets *et al.* (2021) considered combined operations for the production of screw blanks, but did not focus on optimising the geometry of the strip cross-section. This work, like the presented study, is aimed at analysing metal deformation processes, but from different perspectives: M. Pylypets *et al.* (2021) focus on combined operations that include several processing stages, while the presented approach offers an analytical description of the rolling process for a trapezoidal strip. Unlike the rolling diagrams shown in Figures 2 and 4, where the sides of the ring are idealised as straight lines, in practice they will have a curved shape. This can be avoided by creating rolls with a profile that matches the cross-sectional profile of the ring after rolling. This approach was not discussed in detail in the works of L. Liang *et al.* (2021; 2022), which focused on the accuracy criteria for rolling rings with a profiled cross-section. The results obtained allow not only to calculate the parameters of the strip for forming rings, but also to optimise the profile of the rolls, which increases the accuracy and quality of the finished products.

Conclusions

When rolling a straight strip between cylindrical or conical rolls, plastic deformation of the metal occurs, i.e. its redistribution in volume. There is no accurate analytical description of such redistribution due to the complexity of the process. This is due to the non-linearity of the deformation characteristics of the material and its interaction with the tool. The proposed analytical description of the metal mass redistribution process is based on two fundamental properties of plastic deformation: volume constancy and material isotropy. These assumptions made it possible to obtain analytical dependencies that link the geometric parameters of the initial strip with the characteristics of the formed ring. The work provides analytical expressions for the elongation coefficient p and the ring radius r, which allow the parameters of the strip for forming rings with different radii to be calculated. A model of step-by-step rolling has been developed, which describes the gradual reduction of the aspect ratio of the trapezoidal cross-section of the strip to a rectangular one. This allows rings with uniform thickness to be obtained and the geometry to be controlled at intermediate stages of deformation, which minimises the risk of defects such as corrugation or tears. Generalised expressions for calculating the parameters of the strip and ring are proposed, which can be applied to different geometric configurations. This allows solving both the direct problem (determining the parameters of the ring for known strip dimensions) and the inverse problem (selecting the parameters of the strip to obtain a ring of a given radius). It is shown that the elongation coefficients p for different rolling cases (with cylindrical and conical rolls) do not depend on the strip width L_0, but only on the aspect ratio H_0 and H_1. This simplifies calculations and allows the same analytical dependencies to be used for different types of strips. The proposed analytical description provides data for the initial adjustment of the rolls, which significantly reduces the time required for equipment preparation and reduces the cost of experimental verification. In the future, the adjustment of the rolls can be refined experimentally, but the availability of analytical dependencies allows this process to be significantly optimised. Prospects for further research lie in the development of an analytical description of obtaining parts by rolling with a variable cross-section, as well as in studying the influence of different materials and rolling conditions on the accuracy of ring forming. This will expand the scope of application of the results obtained and increase their versatility for industrial applications.

Acknowledgements

The authors of the article express their gratitude to the defenders of Ukraine, whose bravery and selflessness enable scientists to continue their work in their home country.

Funding

None.

Conflict of Interest

None.

References

[1] Bai, D., Gao, P., Yan, X., & Wang, Y. (2021). Intelligent forming technology: State-of-the-art review and perspectives. *Journal of Advanced Manufacturing Science and Technology*, 1(3), article number 2021008. doi: 10.51393/j.jamst.2021008.

[2] Caoqi, Y., Xuedao, S., Jitai, W, Yingxiang, X., Zixuan, L., & Haijie X. (2024). The influence of process parameters on the axial force of the continuous rolling section in the synchronous forming process of shape and inner hole for hollow axles. *Materials Research Proceedings*, 44, 519-527. doi: 10.21741/9781644903254-56.

[3] Chang, X., Fu, W., Li, M., Wang, X., & Yang, W. (2023). The effects of central cross-section diameters of rollers on doubly curved surface rolling. *The International Journal of Advanced Manufacturing Technology*, 126(7-8). doi: 10.1007/s00170-023-11295-3.

[4] Dong, Z., Peng, L., Jiao, Y., Sun, J., Zhuo, C., Zhao, J., Lu, M., Yang, Z., & Wang, P. (2019). Rolling force model of large conical cylinders considering conical metal flow. *China Mechanical Engineering*, 30(19), 2385-2393. doi: 10.3969/j.issn.1004-132X.2019.19.017.

[5] Han, X., Min, Ya., Wuhao, Zh., Hua, L., Tian, D., Deng, Z., & Liu, N. (2023). Radial envelope forming mechanism and process design method for cylindrical rings with thin wall and high web ribs. *Chinese Journal of Aeronautics*, 36(12), 461-476. doi: 10.1016/j.cja.2023.07.024.

[6] Hevko, I.B., Dovbush, T.A., Tsion, O.P., Dovbush, A.D., & Stanko, A.I. (2021). Synthesis of screw working bodies with elastic surfaces and the results of their research. *Agricultural Machines*, 47, 63-72. doi: 10.36910/acm.vi47.649.

[7] Hua, L., Tian, D.Y., Han, X.H., & Zhuang, W. (2023). Modelling and analysis of metal flowing behaviors in constraining ring rolling of tapered ring with thin wall and three high ribs. *Chinese Journal of Aeronautics*, 36(6), 476-492. doi: 10.1016/j.cja.2023.04.011.

[8] Klendii, M.B., & Dragan, A.P. (2021). Justification of the design of the working body of the screw section of the combined soil-cultivating tool. *Perspektyvni Tekhnolohii ta Prylady*, 18, 66-73.

[9] Liang, L., Guo, L., Liu, Z., Wang, P., & Zhang, H. (2021). On a precision forming criterion for groove-section profiled ring rolling process. *Journal of Materials Processing Technology*, 296, article number 117207. doi: 10.1016/j.jmatprotec.2021.117207.

[10] Liang, L., Guo, L., Yang, J., & Zhang, H. (2022). Formation mechanism and control method of multiple geometric defects in conical-section profiled ring rolling. *Journal of Materials Processing Technology*, 306, article number 117628. doi: 10.1016/j.jmatprotec.2022.117628.

[11] Liashuk, O.L., Diachun, A.Ye., Tretiakov, O.L., Navrotska, T.D., & Kruhlik, O.A. (2019). Technical and economic justification of the process of manufacturing screw working bodies. *Visnyk Kharkivskoho Natsionalnoho Tekhnichnoho Universytetu Silskohospodarstva imeni Petra Vasylenka. Mekhanizatsiya Silskohospodarskoho Vyrobnytstva*, 198, 244-251.

[12] Miao, J., Wang, Sh., Shan, X., & Chen, B. (2022). Investigation on contact behavior of planetary roller screw mechanism considering thermal deformation. *Transactions of The Canadian Society for Mechanical Engineering*, 47(1). doi: 10.1139/tcsme-2022-0044.

[13] Nesvidomin, A., Pylypaka, S., Volina, T., Shtyka, Yu., & Rybenko, I. (2025). Optimisation of a developable surface model passing through a helical curve with variable pitch. *Machinery & Energetics*, 16(2), 49-57. doi: 10.31548/machinery/2.2025.49.

[14] Oh, I.Y., Hwang, T.W., Woo, Y.Y., Yun, H.J., & Moon, Y.H. (2019). Process-induced defects in an L-shape profile ring rolling process. *International Journal of Material Forming*, 12, 727-740. doi: 10.1007/s12289-018-1450-3.

[15] Pater, Z., Tomczak, J., Lis, K., Bulzak, T., & Shu, X.D. (2020). Forming of rail car axles in a CNC skew rolling mill. *Archives of Civil and Mechanical Engineering*, 20(3), article number 69. doi: 10.1007/s43452-020-00075-5.

[16] Pylypaka, S.F., Kresan, T.A., Khropost, V.I., & Babka, V.M. (2022). Features of bending a blank in the form of a flat ring into a helical conoid. *Prykladna Heometriya ta Inzhenerna Hrafika*, 102, 157-164. doi: 10.32347/0131-579X.2022.102.157-164.

[17] Pylypets, M.I., Vasylkiv, V.V., Radyk, D.L., & Pylypets, O.M. (2021). Preconditions for the development of combined operations for the production of screw and auger blanks by metal pressure processing. *Perspektyvni Tekhnolohii ta Prylady*, 18, 112-123. doi: 10.36910/6775-2313-5352-2021-18-17.

[18] Qian, D., Gu, Y., Deng, J., & Wang, F. (2024). Deformation laws and design method of near-net rolling for L-section rings with profile axial conical rolls. *Materials Research Proceedings*, 44, 320-329. doi: 10.21741/9781644903254-34.

[19] Qian, D.S., Tian, H., & Deng, J.D. (2023). Towards extremely large-scale radial-axial ring rolling for constant ring growth state with accurate closed-loop control method. *Journal of Mechanical Engineering*, 59, 85-95. doi: 10.3901/JME.2023.10.085.

[20] Yuan, S.J., & Fan, X.B. (2019). Development and perspectives on the precision forming processes for ultra-large size integrated components. *International Journal of Extreme Manufacturing*, 1(2), article number 022002. doi: 10.1088/2631-7990/ab22a9.

[21] Yuen, W.Y.D. (2025). An analysis of the strip deformation and thermal variations in the roll bite for flat rolling. In *Proceedings of the ASME 1995 international mechanical engineering congress and exposition. recent advances in heat transfer and micro-structure modelling for metal processing.* (pp. 57-66). San Francisco, USA. doi: 10.1115/IMECE1995-0967.

Формування смуги із трапецеїдальним поперечним перерізом у кільце прокаткою між конічними валками

Тетяна Кресан
Кандидат технічних наук, доцент
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0002-8280-9502
Тетяна Воліна
Доктор технічних наук, доцент
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0001-8610-2208
Віктор Несвідомін
Доктор технічних наук, професор
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0002-1495-1718
Тетяна Федорина
Кандидат педагогічних наук, доцент
Національний університет біоресурсів і природокористування України
03041, вул. Героїв Оборони, 15, м. Київ, Україна
https://orcid.org/0000-0002-8854-3768
Тарас Пилипака
Кандидат технічних наук, доцент
Національний університет водного господарства та природокористування
33028, вул. Соборна, 11, м. Рівне, Україна
https://orcid.org/0009-0000-5582-1859

Анотація. Для формування гвинтових поверхонь потрібне кільце, що дає змогу виготовляти їх неперервно та без зварних швів. Метою дослідження було розроблення аналітичного опису процесу деформації прямолінійної смуги у кільце в результаті її прокатки між конічними валками. Методологія дослідження ґрунтувалася на аналітичному моделюванні процесу пластичної деформації смуги із використанням припущень про ізотропність матеріалу та незмінність його об'єму. У результаті дослідження отримано аналітичні залежності між розмірами поперечного перерізу смуги до прокатки та параметрами кільця після прокатки, що дозволяють визначати оптимальні геометричні характеристики для формування кілець заданого радіуса. Розроблено узагальнені аналітичні вирази для визначення коефіцієнта подовження та радіуса кільця, які враховують зміну співвідношення сторін трапецеїдального перерізу смуги. Запропоновано модель поетапної прокатки, яка описує поступове зменшення співвідношення сторін трапецеїдального перерізу до прямокутного, що забезпечує отримання рівномірного поперечного перерізу кільця. Отримані вирази дали можливість розв'язувати як пряму, так і обернену задачі, визначати параметри вихідної смуги для формування кілець різного радіуса, прогнозувати геометрію кільця за відомими розмірами смуги, а також використовувати отримані дані для налаштування валків перед експериментальною перевіркою. Запропонована модель поетапної прокатки забезпечила рівномірний поперечний переріз кільця, що підвищує точність виготовлення гвинтових поверхонь без зварних швів. Результати можуть бути використані у машинобудуванні для виготовлення шнеків, гвинтових поверхонь і транспортерів без зварних швів, зокрема в сільськогосподарській, харчовій, будівельній та енергетичній галузях

Ключові слова: пластична деформація; об'єм; ізотропний матеріал; радіус кільця; відносне подовження