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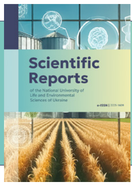

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Development of a mathematical model of stabilisation of device for small-sized cargo transportation

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Abstract. The relevance of the study is conditioned by the need to improve the efficiency and safety of transportation of small-sized cargo. The purpose of this study was to build a mathematical model of the dynamics of stabilisation of device for small-sized cargo transportation. For this, the equations of motion of the system were formulated in the form of a system of second order Lagrange differential equations of the second kind. A grey box approach was used to determine the unknown coefficients of the equations of motion. To implement the approach, an optimisation criterion was constructed that reflected the parameters of the root-mean-square and maximum absolute errors of the differences between theoretical and experimental data of the tilt angle and angular velocity of the device. A modified Ring-Rot-PSO particle swarm method was used to minimise the criterion. The unknown parameters of the device model were found, and the adequacy of the obtained mathematical model was assessed by individual components of the criterion, which proved the adequacy of the obtained mathematical model. To find the unknown parameters, namely the coefficients of the equation of

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motion of the device, a grey box approach was applied. For this, experimental studies of the device stabilisation were performed, and the difference function was formed as an objective function of theoretical, obtained based on analytical equations of motion and experimental data. The objective function was minimised using the modified particle swarm method Ring-Rot-PSO. As a result of the optimisation, the unknown parameters of the system were obtained: the moments of inertia of the frame $I_{1c} = 5.52 \cdot 10^{-4}$ kg·m² and the wheel $I_{wc} = 2.75 \cdot 10^{-3}$ kg·m², the wheel mass $m_w = 3.31 \cdot 10^{-1}$ kg. These data allowed obtaining an adequate mathematical model model of the stabilisation of the device, which underlies further solving of the problem of synthesising optimal control of its motion

Keywords: two-wheeled vehicle; equations of motion; motion control; unstable dynamic system; Ring-Rot-PSO

Introduction

Transportation of cargo forms an integral part of any commodity production, the delivery of goods, and everyday life. Small-sized automated systems for the transportation and courier delivery of small-sized cargo, goods, and products are becoming increasingly widespread. The designs of such devices are represented by both aircraft and ground wheel systems. One of the key tasks of developing such devices is to synthesise optimal control of their motion, which requires the development of an adequate mathematical model of the motion of such devices. This study is dedicated to this issue.

Stable configurations of four to six wheels are commonly used as wheel propulsion systems. Two-wheeled devices are represented by systems where the wheels are on the same axis of rotation. Such devices cannot stabilise their position in static conditions.

The dynamics of ensuring the stability of two-wheeled devices (such as a scooter) has been understudied. To maintain transverse stability in static, such devices require additional mechanisms, such as gyroscopes (Lin *et al.*, 2018), jet wheels (Hongyang & Ruizhi, 2020), etc.

In any case, the synthesis of control systems for such devices is based on adequate mathematical models (Aulin *et al.*, 2024). Thus, the synthesis of optimal motion control of a device for transporting small-sized cargo, which is a non-linear dynamic system, necessitates the construction of an adequate mathematical model. To obtain such a mathematical model, the problem of measuring the dynamic parameters of the system with a given accuracy arises (Rogovskii, 2021). Such measurements are not always possible, and therefore the problem of estimating such parameters based on experimental data obtained during the stabilisation of the device position becomes relevant.

Usually, scientists use the classical method of developing a mathematical model, including finding the parameters of the device, building a dynamic model, and designing of differential equations of motion. Specifically, V. Mudeng *et al.* (2020) describes a method for constructing a mathematical model of a two-wheeled device, such as a segway, which is an unstable dynamic "inverse pendulum" system. To build the mathematical model, the equilibrium method was used, which is based on the d'Alembert's principle of dynamic equilibrium. Some geometric and dynamic characteristics were specified, while others were found theoretically using a computer model of the system. The results of the test were performed only theoretically, without verification on a physical model of the device.

J. Velagic *et al.* (2021) describe a method for constructing a mathematical model using second order Lagrange equations for a two-wheeled device such as a segway. The dynamic characteristics of the device were determined approximated from the kinematic diagram of the device. As a result, the model was used to select the coefficients of the PD controller and an experiment was conducted to assess the quality of the model. There were significant discrepancies between the theoretical and experimental data on the position stabilisation of the device. The researchers explained this discrepancy by referring to inaccuracies in the definition of the dynamic parameters of the device.

M.A. Tofigh *et al*. (2021) describe a two-wheeled device that works on a principle comparable to that of a reverse pendulum. However, two gyroscopes provide stability. The composite mathematical model of the device is presented in the form of second order Lagrange equations. The parameters of the device were determined theoretically using its idealised computer model. O. Obadina *et al*. (2022) described the application of the "grey box" and "white box" approaches and their comparison with experimental data on the device's performance. Thus, the unknown parameters of the model were found from the experimental data of the device operation by applying an optimisation criterion containing the root mean square error values. The criterion was minimized using the GWO-WOA optimization algorithm. This resulted in an error that, compared to the conventional "white box" approach, was an order of magnitude smaller than the one obtained based on theoretically determined values of the model's dynamic parameters. E. Cachaya *et al*. (2024) investigated the stabilisation of the position of a two-wheeled device with a balancing mechanism in the form of a jet wheel. The mathematical model was developed using the equilibrium method. It was later used to optimize the parameters of controllers.

D. Vasilevski *et al*. (2023) considered the problem of synthesising optimal control of the position stabilisation of a two-wheeled device such as a motorcycle, where the balancing mechanism is implemented in the form of a jet wheel. To solve the problem, a mathematical model with two degrees of freedom was proposed, the kinematic scheme of which was presented in the form of an inverse pendulum with a flywheel. The mathematical model of the device was represented in the state space of the system. The dynamic parameters of the system were determined by theoretical methods. M. Horoub *et al*. (2023) considered the mechanism of balancing a two-wheeled vehicle such as a bicycle using propeller thrust. A two-stage process of concept development and testing was presented, first on a simple model and then on a full-scale vehicle. The result was a mathematical model in the form of a matrix of system states. The dynamic parameters of the system were determined approximately. As a result, when checking the adequacy of the model in comparison with the experiment, the model poorly describes the damped self-oscillations when the position of the device changes.

The purpose of the present study was to obtain a mathematical model of the dynamics of the position stabilisation of a device for small-sized cargo transportation.

To fulfil this purpose, the following objectives had to be met:

1) to build a kinematic scheme of the device in the mode of stabilisation of its position and, on its basis, to wright the equations of motion of the device in the form of a system of second order Lagrange equations;

2) to develop an optimization criterion that reflects the difference between theoretical and experimental data on the angle and angular velocity of the device as a function of unknown parameters of the equation of motion of the device and find its minimum by using the Ring-Rot-PSO algorithm;

3) to analyse the quality of the developed mathematical model.

Materials and Methods

To build a mathematical model of a device for small-sized cargo transportation, a kinematic scheme of the device's operation in the balancing mode was presented (Fig. 1). When building the model, a partial mode of operation was considered, namely balancing the device, provided that the wheels of the device do not rotate.

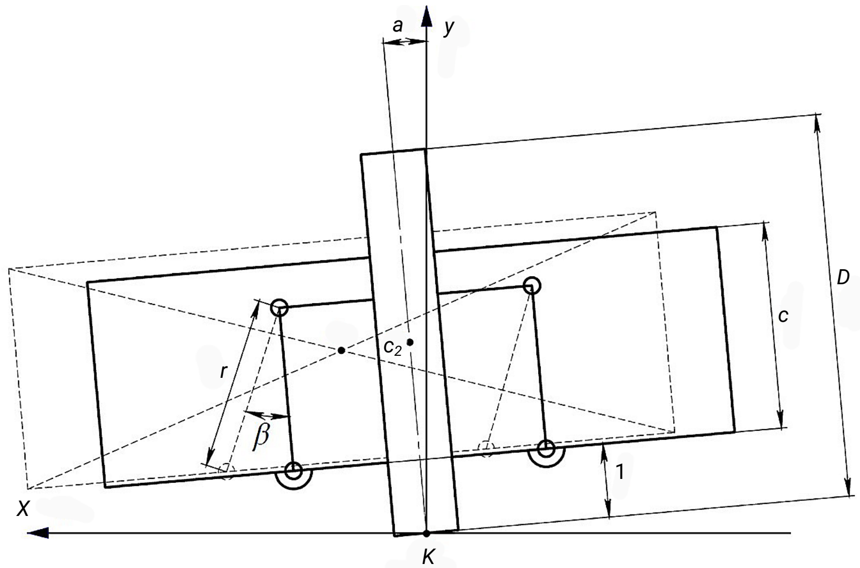

Figure 1. *Kinematic scheme of the device in balancing mode*

Note: α is the angle of inclination of the device in the vertical plane; β is the angle of displacement of the crank of the balancing mechanism from the initial position; c_1 and c_2 is the centre of mass of the frame and the rear wheel, respectively; r is the radius of the crank; c is the frame height; D is the wheel height; l is the distance from the frame to the point of contact K of the wheel with the ground

Source: compiled by the authors of this study

Assumptions were made in developing the model:

1) the frame and rear wheel have an even weight distribution;

2) the centres of gravity of the frame and wheels coincide with the geometric centres of their profiles (marked respectively by points c_1 and c_2);

3) the wheel is in contact with the ground at a point K.

To build a mathematical model of the device, the expression of the kinetic T is compiled:

$$T = \frac{1}{2}\left(\left(I_m + I_p\right) \cdot u^2 + I_p + I_{wc}\right) \cdot \dot{\beta}^2 + \frac{m_1}{2} \cdot \cdot (\dot{x}_{c1}^2 + \dot{y}_{c1}^2) + m_w \cdot \frac{1}{2}(I_{lw} + I_{wc}) \cdot \dot{\alpha}^2, \quad (1)$$

where I_p is the moment of inertia of the pulleys in the belt transmission; I_m is the moment of inertia of the step motor; u is the gear ratio of belt transmission ($u = 1$); I_{wc} is the moment of inertia of the wheel relative to the point of contact with the surface K (Fig. 1); m_1 is the mass of the frame; m_w is the mass of the wheel; I_{lc} is the moment of inertia of the frame relative to the point of contact with the surface K; x_{c1}, y_{c1} are the coordinates of the centre of mass of the device frame. The potential energy of the system is as follows:

$$P = m_1 g y_{c1} + m_k g y_{c2}, \quad (2)$$

where g is the acceleration of free fall; y_{c2} is the coordinate of the centre of mass of the wheel, which is found as follows:

$$y_{c2} = \frac{D}{2} \cos\alpha \approx \frac{D}{2}, \quad (3)$$

where D is the diameter of the wheel. Hereafter, since the angle α varies within a small range (+-5°), we assume $\cos(\alpha) = 1$, $\sin(\alpha) = \alpha$. The coordinates of the centre of mass of the frame are obtained from the following expressions:

$$x_{c1} = r \cdot \sin\beta + y_1 \cdot \sin\alpha, \quad (4)$$

where r is the radius of the crank;

$$y_{c1} = y_1 \cdot \cos\alpha, \quad (5)$$

where y_1 is the height of the centre of the frame under the condition $\alpha = 0$. It is found as follows:

$$y_1 = l + \frac{c}{2} + r \cdot (1 - \cos\beta), \quad (6)$$

where l is the distance from the bottom of the frame to the ground at $\alpha = 0$; c is the height of the frame. The first derivative of y_1 in time is found as follows:

$$\dot{y}_1 = \dot{\beta} \cdot r \cdot \sin\beta, \quad (7)$$

By substituting y_1 in equations (4) and (5), we obtain:

$$x_{c1} = r \cdot \sin\beta + y_1 \cdot \sin\alpha \approx r \cdot \sin\beta + \alpha \cdot \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right), \quad (8)$$

$$y_{c1} = y_1 \cdot \cos\alpha = \cos\alpha \cdot \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right) \approx \approx l + \frac{c}{2} + r \cdot (1 - \cos\beta). \quad (9)$$

The first derivative of the frame's centre of mass coordinates was found as follows:

$$\dot{x}_{c1} = \dot{\beta} \cdot r \cdot \cos\beta - \dot{y}_1 \cdot \sin\alpha + y_1 \cdot \dot{\alpha} \cdot \cos\alpha = = \dot{\beta} \cdot r \cdot \cos\beta - \dot{\beta} \cdot r \cdot \sin\beta \cdot \sin\alpha + \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right) \cdot \dot{\alpha} \cdot \cos\alpha = = \dot{\beta} \cdot r \cdot (\cos\beta - \sin\beta \sin\alpha) + \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right) \cdot \dot{\alpha} \cdot \cos\alpha \approx \approx \dot{\beta} \cdot r \cdot (\cos\beta - \alpha \cdot \sin\beta) + \dot{\alpha} \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right); \quad (10)$$

$$\dot{y}_{c1} = \dot{y}_1 \cdot \cos\alpha - y_1 \cdot \dot{\alpha} \cdot \sin\alpha = = \dot{\beta} \cdot r \cdot \sin\beta \cdot \cos\alpha - \dot{\alpha} \cdot \sin\alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right) \approx \approx \dot{\beta} \cdot r \cdot \sin\beta - \dot{\alpha} \cdot \alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right). \quad (11)$$

To build a mathematical model, the second order Lagrange equation was used as follows:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = -\frac{\partial P}{\partial \alpha}; \quad (12)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\beta}} - \frac{\partial T}{\partial \beta} = M_u \cdot \eta - \frac{\partial P}{\partial \beta}, \quad (13)$$

where M_u is the torque on the step motor shaft; η is the efficiency ratio of the belt transmission ($\eta = 0.9$). To find the partial derivatives of the potential and kinetic energies, the corresponding partial derivatives are taken as follows:

$$\frac{\partial T}{\partial \alpha} = m_1 \left(\dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \alpha} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \alpha}\right); \quad (14)$$

$$\frac{\partial T}{\partial \dot{\alpha}} = m_1 \left(\dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \alpha} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \alpha}\right) + (I_{lc} + I_{wc}) \cdot \dot{\alpha}; \quad (15)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} = m_1 \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \alpha} + \dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \alpha} + + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \alpha} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \alpha}\right) + (I_{lc} + I_{wc}) \cdot \ddot{\alpha}; \quad (16)$$

$$\frac{\partial T}{\partial \beta} = m_1 \left(\dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \beta} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \beta}\right); \quad (17)$$

$$\frac{\partial T}{\partial \dot{\beta}} = I_{cm} \cdot \dot{\beta} + m_1 \left(\dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \dot{\beta}} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \dot{\beta}}\right), \quad (18)$$

where I_{cm} is the combined moment of inertia of the device relative to the point of contact with the ground K (Fig. 1) ($I_{cm} = I_m + I_p + I_c$);

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\beta}} = I_{cm} \cdot \dot{\beta} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \beta} + \dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \beta} + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \beta} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \beta}\right); \quad (19)$$

$$\frac{\partial P}{\partial \alpha} = m_1 \cdot g \cdot \frac{\partial y_{c1}}{\partial \alpha} + m_w \cdot g \cdot \frac{\partial y_{c2}}{\partial \alpha}; \quad (20)$$

$$\frac{\partial P}{\partial \beta} = m_1 \cdot g \cdot \frac{\partial y_{c1}}{\partial \beta} + m_w \cdot g \cdot \frac{\partial y_{c2}}{\partial \beta}. \quad (21)$$

Substituting the results of (14)-(21) into expressions (12) and (13), the following system of differential equations is obtained:

$$
\begin{cases}
I_{cm} \cdot \ddot{\beta} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \beta} + \dot{x}_{c1} \cdot \frac{\partial \dot{x}_{c1}}{\partial \beta} + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \beta} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \beta}\right) - m_1 \cdot \left(\dot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \beta} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \beta}\right) = \\
= M_u \cdot \eta - m_1 \cdot g \cdot \frac{\partial y_{c1}}{\partial \beta} + m_w \cdot g \cdot \frac{\partial y_{c2}}{\partial \beta}; \\
(I_{lc} + I_{wc}) \cdot \ddot{\alpha} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \alpha} + \dot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \alpha} + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \alpha} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \alpha}\right) - m_1 \cdot \left(\dot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \alpha} + \dot{y}_{c1} \cdot \frac{\partial \dot{y}_{c1}}{\partial \alpha}\right) = \\
= -m_1 \cdot g \cdot \frac{\partial y_{c1}}{\partial \alpha} - m_w \cdot g \cdot \frac{\partial y_{c2}}{\partial \alpha}.
\end{cases} \quad (22)
$$

Having opened the brackets and performing simplifications in the system of equations (22), the following system of equations is obtained:

$$
\begin{cases}
I_{cm} \cdot \ddot{\beta} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \beta} + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \beta}\right) = M_u \cdot \eta - m_1 \cdot g \cdot \frac{\partial y_{c1}}{\partial \beta} + m_w \cdot g \cdot \frac{\partial y_{c2}}{\partial \beta}; \\
(I_{lc} + I_{wc}) \cdot \ddot{\alpha} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \alpha} + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \alpha}\right) = -m_1 \cdot g \cdot \frac{\partial y_{c1}}{\partial \alpha} - m_w \cdot g \cdot \frac{\partial y_{c2}}{\partial \alpha}.
\end{cases} \quad (23)
$$

To find the unknown parts of (23), consider the following:

$$
\frac{\partial x_{c1}}{\partial \alpha} = \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) cos\,\alpha \approx \\
\approx l + \frac{c}{2} + r \cdot (1 - cos\,\beta); \quad (24)
$$

$$
\frac{\partial y_{c1}}{\partial \alpha} = \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) sin\,\alpha \approx \\
\approx -\alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right); \quad (25)
$$

$$
\frac{\partial y_{c2}}{\partial \alpha} = \frac{D}{2} sin\,\alpha \approx \frac{D}{2} \alpha; \quad (26)
$$

$$
\frac{\partial y_{c2}}{\partial \beta} = r \cdot cos\,\beta + r \cdot sin\,\beta\, sin\,\alpha \approx \\
\approx r \cdot (cos\,\beta + \alpha \cdot sin\,\beta); \quad (27)
$$

$$
\frac{\partial y_{c1}}{\partial \beta} = r \cdot sin\,\beta\, cos\,\alpha \approx r \cdot sin\,\beta; \quad (28)
$$

$$
\frac{\partial y_{c2}}{\partial \beta} = 0. \quad (29)
$$

To find the second derivative for the coordinate x_{c1} of the centre of mass of the frame, the following expression is formulated:

$$
\ddot{x}_{c1} = \beta \cdot r \cdot (cos\,\beta - sin\,\beta \cdot sin\,\alpha) - \beta \cdot r \cdot \left(\dot{\beta} \cdot sin\,\beta - \beta \cdot cos\,\beta\, sin\,\alpha - \dot{\alpha} \cdot sin\,\beta\, cos\,\alpha\right) + \\
+ \ddot{\alpha} \cdot cos\,\alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) + \dot{\alpha} \cdot \left(\dot{\beta} \cdot r \cdot sin\,\beta\, cos\,\alpha - \dot{\alpha} \cdot sin\,\alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right)\right). \quad (30)
$$

The brackets are opened, and the equation is simplified:

$$
\ddot{x}_{c1} = \ddot{\beta} \cdot r \cdot (cos\,\beta - \alpha \cdot sin\,\beta) - \dot{\beta} \cdot r \cdot \left(\dot{\beta} \cdot (sin\,\beta - \alpha \cdot cos\,\beta) - \dot{\alpha} \cdot sin\,\beta\right) + \\
+ \ddot{\alpha} \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) + \dot{\alpha} \cdot \left(\dot{\beta} \cdot r \cdot sin\,\beta - \dot{\alpha} \cdot \alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right)\right); \quad (31)
$$

$$
\ddot{x}_{c1} \approx \ddot{\beta} \cdot r \cdot (cos\,\beta - \alpha \cdot sin\,\beta) + \ddot{\alpha} \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) - \dot{\beta}^2 \cdot r \cdot (sin\,\beta - \alpha \cdot cos\,\beta) - \\
- \dot{\alpha}^2 \cdot \alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) + 2\dot{\alpha} \cdot \dot{\beta} \cdot r \cdot sin\,\beta. \quad (32)
$$

The second derivative of the coordinate y_{c1} of the centre of mass of the frame is determined:

$$
\ddot{y}_{c1} = \beta \cdot r \cdot sin\,\beta \cdot cos\,\alpha + \beta \cdot r \cdot \left(\dot{\beta} \cdot cos\,\beta \cdot cos\,\alpha - \dot{\alpha} \cdot sin\,\beta\, sin\,\alpha\right) - \\
- \ddot{\alpha} \cdot sin\,\alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right) - \dot{\alpha} \cdot \left(\dot{\beta} \cdot r \cdot sin\,\beta\, sin\,\alpha - \dot{\alpha} \cdot cos\,\alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - cos\,\beta)\right)\right). \quad (33)
$$

Having performed the transformation, let us reduce expression (33) to the following form:

$$\ddot{y}_{c1} \approx \ddot{\beta} \cdot r \cdot \sin\beta - \ddot{\alpha} \cdot \alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right) + \dot{\beta}^2 \cdot r \cdot \cos\beta$$
$$-2\dot{\alpha} \cdot \dot{\beta} \cdot \alpha \cdot r \cdot \sin\beta - \dot{\alpha}^2 \cdot \alpha \cdot \left(l + \frac{c}{2} + r \cdot (1 - \cos\beta)\right). \quad (34)$$

Substituting all the results obtained into the system of equations (20), a system of differential equations is obtained:

$$\begin{cases} I_{cm} \cdot \ddot{\beta} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \beta} + \ddot{y}_{c1} \cdot \frac{\partial y_{c1}}{\partial \beta}\right) = M_u \cdot \eta - \left(m_1 \frac{\partial y_{c1}}{\partial \beta} + m_w \frac{\partial y_{c2}}{\partial \beta}\right) \cdot g; \\ (I_{lc} + I_{wc}) \cdot \ddot{\alpha} + m_1 \cdot \left(\ddot{x}_{c1} \cdot \frac{\partial x_{c1}}{\partial \alpha} + y_{c1} \cdot \frac{\partial y_{c1}}{\partial \alpha}\right) = -\left(m_1 \frac{\partial y_{c1}}{\partial \alpha} + m_w \frac{\partial y_{c2}}{\partial \alpha}\right) \cdot g, \end{cases} \quad (35)$$

The system of differential equations (35) is a mathematical model of the dynamics of balancing a device for small-sized cargo transportation.

In model (35), the coefficients in equations (35) are known, but some need to be determined, i.e., identified. This allows obtaining an adequate mathematical model that would correspond to the dynamics of device stabilisation. An adequate mathematical model underlies the synthesis of a controller that would ensure the optimised stabilisation of the device.

Results and Discussion

The individual device parameters included in equation (35) can be easily measured. Their numerical values are given in Table 1.

Table 1. *Numerical values of known parameters of a device for small-sized cargo transportation*

Parameter name	Unit of measurement	Symbol	Numeric value
Frame mass	kg	m_1	9.5
Crank radius	m	r	0.047
Distance from ground to the frame	m	l	0.040
Frame height	m	c	0.080
Wheel diameter	m	D	0.136

Source: developed by the authors of this study

Since the stabilisation process of the device is controlled by the angular velocity $\dot{\beta}$ of the balancing mechanism, let us consider the second equation of the system (35), which includes the desired (unknown) parameters of the device. They were found by using experimental data from the device by using "grey box" approach (Romlay *et al.*, 2019; Komor *et al.*, 2020).

The experimental data were collected in the stabilisation mode of the device in the form of an array of numerical data. The array data was collected with a time step of $\Delta t = 0.006$ s. The duration of data acquisition when the device was in stabilisation mode was 15 s. At each time interval, the following were read: tilt angle α, angular velocity of the device tilt $\dot{\alpha}$, angular velocity of the balancing mechanism rotation $\dot{\beta}$, and the current program operation time t.

The stability of the device was achieved using a proportional-differential controller (PD controller), the coefficients of which were selected empirically:

$$\dot{\beta} = 3 \cdot a - 3 \cdot \dot{\alpha}. \quad (36)$$

Accordingly, the values of the slope angle β and its second-time derivative were found in the components of equation (35):

$$\beta = \int (3 \cdot \alpha - 3 \cdot \dot{\alpha})\, dt; \quad (37)$$

$$\ddot{\beta} = 3 \cdot \dot{\alpha} - 3 \cdot \ddot{\alpha} \quad (38)$$

Having made all the substitutions, the second equation of the system (35) takes the following expanded form:

$$\begin{aligned} &2 \cdot g \cdot \alpha \cdot (c \cdot m_1 - D \cdot m_w + 2 \cdot m_1 \cdot (l + r)) + 6 \cdot m_1 \cdot r^2 \times \\ &\times sin(6 \cdot \int (\alpha + \dot{\alpha}) \cdot \mathrm{d}t) \cdot (\alpha + \dot{\alpha}) \cdot (3 \cdot \alpha + 5 \cdot \dot{\alpha}) = \\ &= 4 \cdot g \cdot \alpha \cdot m_1 \cdot r \cdot cos(3 \cdot \int (\alpha + \dot{\alpha})\, dt) + 6 \cdot m_1 \cdot r \cdot \\ &\cdot (c + 2 \cdot (l + r)) \times sin(3 \cdot \int (\alpha + \dot{\alpha})\, dt) \cdot (\alpha + \dot{\alpha}) \cdot \\ &\cdot (3 \cdot \alpha + 5 \cdot \dot{\alpha}) + 4 \cdot \ddot{\alpha} \cdot (I_{lc} + I_{wc}) + m_1 \cdot (c + 2 \cdot (l + r) - \\ &- 2 \cdot r \cdot cos(3 \cdot \int (\alpha + \dot{\alpha}) \cdot dt)) \times (\ddot{\alpha} \cdot (c + 2 \cdot (l + r)) - \\ &- 2 \cdot r \cdot cos(3 \cdot \int (\alpha + \dot{\alpha}) \cdot d\mathrm{t}) \cdot (3 \cdot \dot{\alpha}) + 4 \cdot \ddot{\alpha})). \quad (39) \end{aligned}$$

To find the solution to equation (39), the initial conditions were set as follows: the tilt angle of the device α_0=-0.033 rad, the angular velocity of the tilt $\dot{\alpha}$= 0.109 rad·s^{-1}. The sought parameters of the device are as follows: 1) moment of inertia of the wheel I_{wc}; 2) moment of inertia of the frame I_{lc}; 3) mass of the wheel m_w. The optimisation criterion *J*, which corresponds to the degree of deviation of theoretical data (obtained using model (39)) from experimental data, is the following expression:

$$J = E_{RMS\alpha} + E_{RMSd\alpha} \cdot T + E_{max\alpha} + E_{maxd\alpha} \cdot T, \quad (40)$$

where $E_{RMS\alpha}$ is the root-mean-square value of the difference between the deviations of theoretical and experimental data by angle α; $E_{RMSd\alpha}$ is the root-mean-square value of the difference between the deviations of theoretical and experimental data in terms of angular velocity $\dot{\alpha}$; *T* is the period of forced oscillations of the device during the experiment (*T* = 0.744 s); $E_{max\alpha}$ is the maximum value of the difference between the deviations of the theoretical data and the experimental data at the angle α; and $E_{maxd\alpha}$ is the maximum error value of the difference between the deviations of theoretical and experimental data in terms of angular velocity $\dot{\alpha}$. In formula (40), the $E_{RMS\alpha}$ value was found as follows:

$$E_{RMS\alpha} = \sqrt{\frac{1}{2500} \cdot \sum_{j=1}^{2500} E_{\alpha.j}^2}\ , \quad (41)$$

where $E_{\alpha.j}$ is the *j*th difference between theoretical data and experimental data for the angle α:

$$E_{\alpha.j} = \alpha_{t\_j} - \alpha_{e\_j}, \quad (42)$$

where α_{t_j} is the *j*th value of the inclination angle α obtained theoretically (solution of equation (39)); α_{e_j} is the *j*th value of the angle α of the array of experimental data. In formula (40), the root-mean-square value of the angular velocity error $E_{RMSd\alpha}$was found as follows:

$$E_{RMSd\alpha} = \sqrt{\frac{1}{2500} \cdot \sum_{j=1}^{2500} E_{d\alpha\_j}^2}, \quad (43)$$

where $E_{d\alpha.j}$ is the *j*th difference between theoretical data and experimental data in terms of angular velocity $\dot{\alpha}$:

$$E_{d\alpha.j} = \dot{\alpha}_{t.j} - \dot{\alpha}_{e.j} \quad (44)$$

where $\dot{\alpha}_{t,j}$ is the *j*th value of the angular velocity , which is obtained theoretically (the first-time derivative of the solution of equation (39)); $\dot{\alpha}_{e\_j}$ is the *j*th value of the angular velocity $\dot{\alpha}$ array of experimental data.

To reduce the elements of the expression to one dimension, the value *T* is used in expression (40). The maximum values of the differences in angles and angular velocities of the theoretical and experimental data $E_{max\alpha}$ and $E_{maxd\alpha}$ are found according to the following formulas:

$$E_{max\alpha} = max(|E_{\alpha\ j}|); \quad (45)$$

$$E_{maxd\alpha} = max(|E_{d\alpha\ j}|). \quad (46)$$

To minimise criterion (40), a modified Ring-Rot-PSO particle swarm method was applied (Romasevych *et al.*, 2021). As a result of its use, numerical values of the parameters were obtained (Table 2).

Table 2. *Numerical values of the found arguments of the criterion J (parameters of the dynamic system)*

Parameter	Dimensions	Numerical value
I_{wc}	kg·m²	5.52·10^{-4}
I_{lc}	kg·m²	2.75·10^{-3}
m_w	kg	3.31·10^{-1}

Source: developed by the authors of this study

The values of the parameters presented in Table 2 allow minimising the value of the criterion *J*, and therefore the difference between the data of the theoretical model (39) and the experimental data is minimal. To analyse the constituent elements of criterion *J*, a plot was built. In Figures 2 and 3, the solid grey curves correspond to theoretical data, the dashed curves – to experimental data, while the red curves – to the difference between theoretical and experimental data.

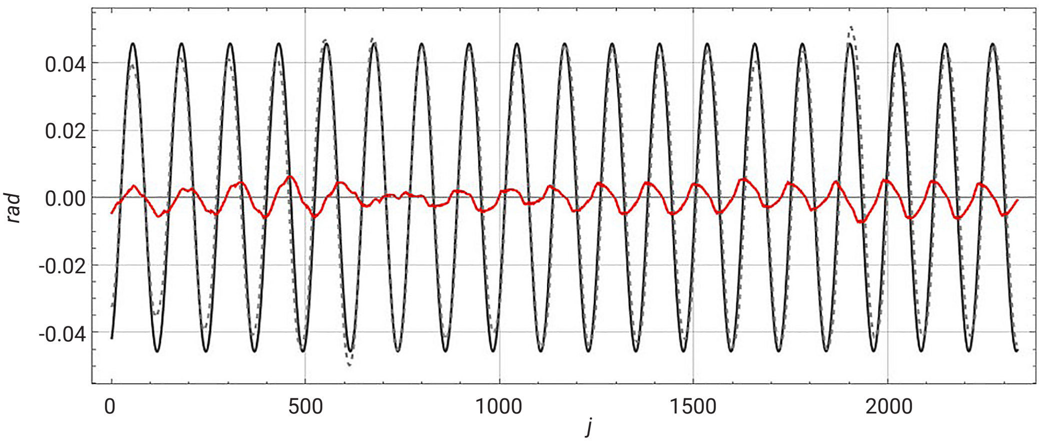

Figure 2. *Plots of the angle of inclination α obtained theoretically and experimentally, as well as their difference*
Source: developed by the authors of this study

Figure 2 shows that the difference between the values of the theoretical and experimental data for the angle α is much smaller than the values of the angle itself (both theoretical and experimental data) and they change almost synchronously. That is, the maximums and minimums of the error plot practically coincide with the maximum and minimum values of the device's tilt angle. The values of the difference between the theoretical and experimental data for the angular velocity $E_{d\alpha}$ (Fig. 3) are much smaller than the values of the experimental and theoretical data themselves. This plot varies proportionally with the values of the angular velocity . Furthermore, the numerical values of the components of the *J* criterion were calculated, which show certain categories of discrepancies between theoretical and experimental data (Table 3)

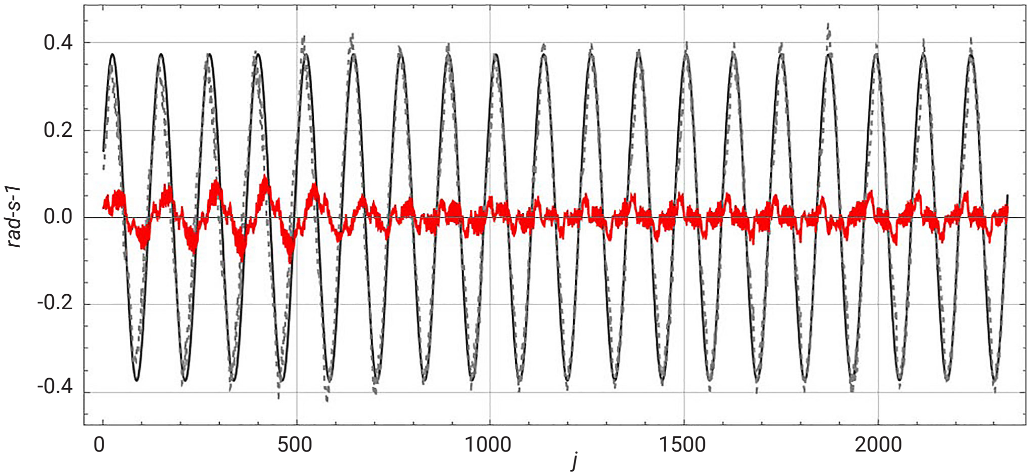

***Figure 3.** Plots of angular velocity , obtained theoretically and experimentally, as well as their difference*

Source: developed by the authors of this study

***Table 3.** The numerical values of the components of the criterion J*

Component of criterion J	Dimensions	Value
$E_{max\alpha}$	rad	$7.31 \cdot 10^{-3}$
$E_{maxd\alpha}$	$rad \cdot s^{-1}$	$9.91 \cdot 10^{-2}$
$E_{RMS\alpha}$	rad	$3.11 \cdot 10^{-3}$
$E_{RMSd\alpha}$	$rad \cdot s^{-1}$	$2.78 \cdot 10^{-2}$

Source: developed by the authors of this study

Considering the values of $E_{RMS\alpha}$ and $E_{RMSd\alpha}$, which are insignificant (Table 3), it can be concluded on the good quality of identification of the parameters of the mathematical model (39). It can then be used to synthesise the optimal motion control for a small cargo transport device in stability mode.

Comparing the results and approach with other researchers, the following characteristics can be obtained. Z. Pluta & T. Hryniewicz (2013) described a method for determining the moment of inertia of the body mass relative to the desired axis experimentally using a physical pendulum with a rigid suspension at the end of which the test body is fixed. The moment of inertia was found by the oscillation parameters (deflection angle, oscillation period, and suspension length). In our case, the moments of inertia of the frame and wheel were found from the experimental data of the device stabilisation by using „grey box" approach, the parameter optimisation criterion, and the Ring-Rot-PSO optimisation algorithm. This allowed simplifying the construction of a mathematical model without the need to build a pendulum system and perform additional calculations of the device's moments of inertia measurements.

B. Rao *et al.* (2022) put the simple physical pendulum method into practice to determine the moment of inertia for a model aircraft. This methodology allows obtaining fairly accurate results: the researchers noted that the error is 1% when comparing theoretical and experimental data of mass inertia moments. This method is not well-suited for the cases described in the present study since the moment of inertia of the frame during the balancing process is not constant. It varies depending on the position of the balancing mechanism, which changes the height of the frame's centre of mass relative to the point of contact of the wheel with the ground, relative to which the frame's moment of inertia is found.

In some cases, it is sufficient to estimate the unknown system parameters, or even to empirically find the required control action to stabilise the position of the device. To ensure the stability in the static of a two-wheeled device (such as a scooter), P. Gogoi *et al*. (2017) used a balancing mechanism in the form of a jet wheel. Its parameters were sought empirically by changing the diameter and weight of the disc, and relevant experiments were conducted to determine the optimised diameter of the jet wheel disc. In our case, the balancing mechanism works on the principle of shifting the mass relative to the centre of mass to ensure static stability. An empirical approach to synthesise the optimal control would take an exceedingly long time and, if the parameters (weight of the cargo, inclination of the road) changed, the balancing would be of questionable quality or even impossible.

L. Nehaoua *et al*. (2013) conducted a thorough study and proposed to use the differential-variational principle of Jourdain to build a mathematical model of a two-wheeled device (such as a motorcycle), accommodating the suspension, elastic properties of the wheels, and the force on the steering wheel. It showed good prediction performance, but this is an example of a theoretical problem without practical verification. In contrast to the Jourdain principle, the present study constructed a mathematical model of the device in the form of second order Lagrange equations, and the use of the „grey box" approach made it possible not to determine the stiffness of the suspension, the elasticity of the wheels, the deformation in the links and the shape of the ground contact patch. These and many other parameters were found from the experimental data of the device.

To ensure the stability of the position of a two-wheeled device (such as a motorcycle) in static and in motion, R. Lot & J. Fleming (2018) proposed to use a gyroscope in two modes (passive and active). The active mode proved to be much better than the passive mode and ensured that the stability was maintained in static. The mathematical model of the device was developed in the form of a system state space. The quality of the device model has not been assessed in practice. This approach requires accurate measurements of the device parameters, which is not always possible. Unlike R. Lot & J. Fleming (2018), the current study is tied to a real device and thus allows not only theoretical verification, but also experimental verification.

Y. Lin *et al*. (2024) considered the problem of maintaining stability in motion for a two-wheeled device (such as a scooter) by controlling the rotation of the front wheel. To solve this problem, a dynamic model of the device's motion was constructed in the form of second order Lagrange equations. The method of determining the dynamic parameters of the device was not described. To ensure stability, the parameters of various variations of the PD controller were found based on the dynamic model. In the cited study, the process of creating a mathematical model partially repeats the methodology presented in the current study, except that the former did not consider ensuring high-quality dynamic parameters of the device. In our case, a more thorough study was carried out, which involved determining the unknown parameters of the model.

M. Khan *et al*. (2022) described a method for constructing a mathematical model of a two-wheeled device (such as a segway) by using the „black box" approach. It is implemented using neural network technologies based on time-series data arrays generated on a known mathematical model of the device. To fulfil this purpose, it was proposed to use two models: simple autoregressive (ARX) and non-linear autoregressive (NLARX). The modelling result showed high quality forecasting with a relative error of less than

0.05%. Compared to the current study, the cited study describes a solution to the second part of the process of creating a mathematical model. It uses experimental data in the form of a time series to determine the dependence of some system parameters on each other.

M. Garziad *et al*. (2024) describe a method for constructing a mathematical model of a two-wheeled device (such as a motorcycle) based on graph theory. In this method, the dynamic model is represented as a system of graphs. The results of the forecasting quality check presented by the researchers, in comparison with other studies, show good prediction quality. Comparing the methodology for creating a mathematical model based on graph theory with the „grey box" approach implemented in the current study, the following differences can be observed. Representation of the dynamic model of the device in the form of graphs and the subsequent process of creating a mathematical model is complex, novel, and requires high qualification of the researcher, unlike the classical methods used in the present study.

Conclusions

A kinematic scheme of a device for small-sized cargo transportation was constructed. A system of differential equations for the dynamics of the device stabilisation was developed by using second order Lagrange equations. To find the unknown parameters (coefficients of the equation of motion of the device), a „grey box" approach was applied. For this, experimental studies of the device stabilisation were conducted, and a difference function (objective function) between theoretical (obtained from the equations of motion) and experimental data was formed. The objective function is minimised using the modified particle swarm method Ring-Rot-PSO. As a result of the optimisation, the unknown parameters of the system were found as follows: moments of inertia of the frame $I_{lc} = 5.52 \cdot 10^{-4}$ kg · m² and wheels $I_{wc} = 2.75 \cdot 10^{-3}$ kg · m², wheel mass $m_w = 3.31 \cdot 10^{1}$ kg. The quality of the mathematical model was evaluated according to the following parameters: the maximum values of the difference between theoretical and experimental data for the angle of inclination of the device $E_{max\alpha} = 7.31 \cdot 10^{-3}$ rad and for the angular velocity $E_{maxd\alpha} = 9.91 \cdot 10^{-2}$ rad·s^{-1}; root mean square values for the tilt angle of the device $E_{RMS\alpha} = 3.11 \cdot 10^{-3}$ rad and for the angular velocity $E_{RMSd\alpha} = 2.78 \cdot 10^{-2}$ rad·s^{-1}. The findings give grounds to consider the obtained mathematical model of stabilisation of the device suitable for further synthesis of optimal control of device position stabilisation. This will enable development of a control algorithm and implement it in software on a prototype device.

The obtained methodology for developing a mathematical model of a device is not limited to this example and can be applied to other dynamic systems. The generality of the methodology is grounded by the use of the classical approach to constructing a dynamic model in the form of Lagrange equations, while the „grey box" approach simplifies the search for unknown system parameters. The findings allow considering this methodology as one of the ways to solve the problem of developing an adequate mathematical model of movement devices such as a segway, scooter, motorcycle, bicycle, and a series of other wheeled unstable mobile platforms. The application of this methodology in other fields of technology and engineering opens new opportunities for the analysis and optimisation of dynamic systems. It will also help to develop more efficient and stable mechanisms in various industries, from transport to robotics.

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Conflict of Interest

None.

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Розробка математичної моделі стабілізації пристрою для транспортування малогабаритних вантажів

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Анотація. Актуальність дослідження обумовлена необхідністю підвищення ефективності та безпеки транспортування малогабаритних вантажів Метою дослідження було побудувати математичну модель динаміки стабілізації пристрою для транспортування малогабаритних вантажів. Для цього складено рівняння руху системи у вигляді системи диференціальних рівнянь Лагранжа другого роду. Для визначення невідомих коефіцієнтів рівнянь руху застосовано підхід «сіра скриня». Для реалізації підходу було побудовано критерій оптимізації, який відображав параметри середньоквадратичних та максимальних абсолютних похибок різниць теоретичних та експериментальних даних кута нахилу та кутової швидкості нахилу пристрою. Для мінімізації критерію було застосовано модифікований метод рою часточок Ring-Rot-PSO. Знайдено невідомі параметри моделі пристрою та проведено оцінку адекватності отриманої математичної моделі за окремими складовими критерію, яка показала адекватність отриманої математичної моделі. Для знаходження невідомих параметрів, а саме коефіцієнтів рівняння руху пристрою, застосовано підхід сіра скриня. Для цього проведено експериментальні дослідження стабілізації пристрою та сформовано функцію відмінності, як цільову функцію, теоретичних, які отримані на основі аналітичних рівнянь руху, і експериментальних даних. Проведено мінімізацію цільової функції за допомогою модифікованого методу рою часточок Ring-Rot-PSO. В результаті оптимізації отримано невідомі параметри системи: моменти інерції рами $I_{1к} = 5{,}52 \cdot 10^{-4}$ кг · м² та колеса $I_{кк} = 2{,}75 \cdot 10^{-3}$ кг · м², масу колеса $m_к = 3{,}31 \cdot 10^{-1}$ кг. Ці дані дозволили отримати адекватну математичну модель стабілізації пристрою, яка є основою для подальшого розв'язання задачі синтезу оптимального керування його рухом

Ключові слова: двоколісний пристрій; рівняння руху; керування рухом; нестійка динамічна система; Ring-Rot-PSO